Research Foundations and Design
Vmath® Second Edition

Cambium LEARNING® Group | Voyager

Table of Contents

Introduction 4
Instructional Design 5
Get Started 5
Try it Together 6
Work On Your Own 6
Check Up 6
Essentials of Math Instruction 7
Conceptual Understanding 7
Fluency in Computation and Operations 7
Problem-Solving 7
Vmath Assessment 8
Initial and Final Assessments 8
Progress Assessments Powered by the Quantile Framework for Mathematics 8
Pre-Tests and Post-Tests 9
Curriculum-Based Measurement 9
Vmath Benchmark and Progress Monitoring Computational Fluency 9
Ongoing Assessment 10
Error Analysis 10
Online Data Management System 10
References 11

Introduction

Many students do not learn effectively from the instruction provided in their regular math classes. They struggle continually with the math concepts, skills, and strategies necessary for success (Cross and Hynes, 1994). These students often fall even further behind as their core program moves on to more difficult material. They score poorly on important tests, attend summer school, and may even be denied advancement to the next grade level. However, examples in the literature (Campbell 1995; Griffin, Case, and Siegler 1994; Knapp et al. 1995; Silver and Stein 1996) demonstrate that all children, including those who have been traditionally underserved, can learn mathematics when they have access to high-quality instructional programs that support their learning. The NCTM Principles and Standards for School Mathematics (PSSM) asserts that students with special educational needs must have the opportunities and support they require to attain a substantial understanding of important mathematics (NCTM, 2000, p. 3). Research, reported by the National Mathematics Advisory Panel, recommends that "struggling students receive some explicit instruction regularly, Some of this time should be dedicated to ensuring that these students possess the foundational skills and conceptual knowledge necessary for understanding the mathematics they are learning at their grade level" (National Mathematics Advisory Panel Final Report, 2008, p. xxiii)

All children can learn mathematics when they have access to high-quality instructional programs that support their learning. (PSSM)

Vmath® is a results-driven, research-based math intervention program for struggling students in grades 2–8. It provides a blended solution of explicit, teacher-led instruction with student-centered technology to address the diverse learning needs of students. Through its balanced approach, instruction focuses on conceptual development and computational fluency as it builds strong problem-solving skills. Guided by the *Vmath* assessments, powered by the Quantile Framework® for Mathematics, teachers provide differentiated intervention to enable all struggling students to be successful. With *Vmath*, students experience improved results and develop the foundational knowledge they need for future success in algebra.

Vmath is designed to complement all major math programs by providing an additional 30-45 minutes of daily, targeted instruction. Each level of *Vmath* contains eight individual modules covering the content strands of mathematics in grades 2-8 including: number operations, algebra, geometry, measurement, and data analysis. The instruction in these modules is aligned with the grade-level expectations of the National Council of Teacher of Mathematics Content Standards of the PSSM (NCTM, 2000) and the NCTM Curriculum Focal Points (NCTM, 2006)

Instructional Design

Vmath is designed using widely accepted principles of effective intervention instruction for struggling students. The direct, systematic instruction in *Vmath* provides carefully sequenced, specific, and detailed dialogue for every lesson. This form of explicit, highly structured lesson delivery uses a direct instruction model based on research by Stein, Silbert, and Carnine (Stein, M., Silbert, J. and Carnine, D., 1997, p. 4).

Nine Steps to Design an Effective Instructional Program

1. **1.** Specify long- and short-term objectives
2. **2.** Devise instructional strategies
3. **3.** Determine necessary pre-skills
4. **4.** Sequence skills
5. **5.** Select a teaching procedure
6. **6.** Design teaching formats
7. **7.** Select examples
8. **8.** Specify practice and review
9. **9.** Design progress-monitoring procedures

Rosenshine (1983), in a review of research on teacher effectiveness, concluded that highly interactive, briskly-paced, clearly presented instruction was related to high rates of student success, and he referred to this type of teaching as direct instruction (explicit, teacher-directive practices). However, the term direct instruction is generally used to refer to the instructional theory work of Engelmann and Carnine (1991). Research provides consistent support for using an explicit approach to teaching mathematics. Adams and Engelmann (1996) analyzed 34 intervention studies and found this approach to be more successful in 32 of the 34 studies they reviewed. Bottge (2001) asserted that teachers should continue to foster competence in basic skills by providing students explicit instruction, and Kroesbergen and Van Luit (2003) reported in a study of 58 research reports that direct instruction was found to be more effective than mediated instruction. The National Mathematics Advisory Panel reported that "Explicit instruction with students who have mathematical difficulties has shown consistently positive effects on performance with word problems and computation. Results are consistent for students with learning disabilities as well as other students who perform in the lowest third of a typical class." (National Mathematics Panel Final Report, 2008, p. xxiii) These studies provide convincing evidence that such pedagogy facilitates the learning process.

Explicit instruction with students who have mathematical difficulties has shown consistently positive effects on performance with word problems and computation.

Explicit instruction is defined by the National Mathematics Advisory Panel as instruction that provides 1) clear models for solving a problem type, 2) opportunities for students to talk through the decisions they make and the steps they take, 3) extensive practice of newly learned strategies and skills, and 4) extensive feedback. *Vmath* instruction follows this instructional model in a clear, four-step lesson format: Get Started, Try it Together, Work On Your Own, and Check Up.

Get Started

Each *Vmath* lesson begins with a review of pre-skills. "Because students learn by connecting new ideas to prior knowledge, teachers must understand what their students already know" (NCTM, PSSM, p.17). The first few problems review the prerequisite skills for the lesson or review the component skills of the process taught later in the lesson. After a review of the preskills, teachers carefully model the new skill or strategy needed to achieve the lesson objective. In Get Started, clear models of the problem type are presented. The instruction is explicit with a script for the teacher to follow. Students are able to connect the skill that is modeled with the underlying mathematical concept using the "Build the Concept" box included on the student page. Since there are no student directions in this section, the lesson script ensures that modeling by the teacher includes everything students need to effectively learn the new skill or strategy.

Try it Together

In the Try it Together section of the *Vmath* lesson, teachers and students talk through the steps

of a process, or the decisions they make in problem-solving, as they practice the new learning together. Teachers continue to provide a clear model as they break the task into manageable chunks.

It is during this section of the lesson that teachers scaffold instruction as they provide temporary support to students as they begin to apply the new skill or strategy. Scaffolded instruction may take different forms including: using questions to guide students through the steps needed to complete a problem, or through the use of graphic organizers. (Stein, M., Kinder, D., Silbert, J., Carnine, D. 2006) Through the use of effective prompts and questions, included in the *Vmath* lesson script, teachers gradually remove the instructional scaffold supporting students as they complete the Try it Together problems. (Hall, 2002) The graphic organizer, called the "How To," box provides both a verbal and symbolic representation of the new concept, skill, or strategy to help students make connections between the language of math and its application.

In Try it Together teachers monitor student performance and use correction procedures as they provide immediate feedback. Teachers verify that students can apply the new concepts, skills, or strategies before moving to the next phase of the lesson, Work On Your Own.

Feedback is important in effective math instruction for all learners.

Work On Your Own

Vmath students practice new and previously learning content independently in the Work On Your Own section of the lesson. If students need continued support, they can refer to the "How To" box to review an explicit example of the new concept, skill, or strategy. Teachers continue to monitor student work and provide corrective feedback as needed. This teaching technique is supported by Smith and Geller (2004) who indicate that feedback is important in effective math instruction for all learners, including learners with disabilities, and those at risk of school failure.

Students keep a Progress Chart of their Work On Your Own achievement in the back of their student book. This practice of self-monitoring has been shown by researchers to enhance academic achievement (Lan, Repman, and Chyung, 1998).

Check Up

After Work On Your Own, students complete the informal assessment, Check Up Students apply what they have learned in a format similar to a high-stakes assessment situation. Three questions target the new learning in the lesson and provide the information teachers need to determine if reteaching is needed. Questions written in multiple choice and short answer formats are an effective and efficient way to check on daily progress. Some of the distracters presented in the multiple choice problems represent common errors made by students. When these specific wrong answers are selected, the teacher uses the error analysis procedures in the side bar to correct the student's misconception or misapplication. Error analysis is another of the common attributes that have been identified as positively affecting student learning (Smith and Geller, 2004).

Vmath provides resources to differentiate instruction and practice for students who need additional support or reteaching. The Reteach Lessons provide a short, alternative method for teaching a new concept or skill, Suggested teaching recommendations are provided for students with special needs and ELL students. *Vmath*Live activities align with select lessons and provide an interactive method for practice and reteaching, if needed. Extra Practice is included in the student book for each *Vmath* lesson. The practice of error analysis with reteaching is aligned with the research findings of Good and Grouws (1979) related to improving learning through reteaching.

Essentials of Math Instruction

The National Mathematics Advisory Panel found that the curriculum must simultaneously develop conceptual understanding, fluency in computation and operations, and problem solving skills. (National Mathematics Panel Final Report 2008) *Vmath* provides a balanced approach of these three essential elements of math instruction with a focus on building a strong foundation in number sense.. A major goal in levels C – F (grades 2 – 5) is the development of computational fluency with whole numbers (NCTM, PSSM, p. 144). Additionally, *Vmath* students reach for computational fluency with fractions and decimal numbers by the end of level G (grade 6). Preparation for algebra begins in the earliest levels of *Vmath* and continues through all the levels.

Conceptual Understanding

Vmath teaches the underlying concepts, or big ideas, needed for understanding mathematics by using hands-on activities, computer simulations, and visual and pictorial representations. Instruction provides the structure by which student understanding transitions from concrete, to representational, to abstract.

Fluency in Computation and Operations

Vmath provides an additional focus on the foundational skills and processes students must master at each level. The "How To" box in each lesson provides clear step-by-step procedures to guide students through the processes needed to complete basic algorithms successfully.

Problem-Solving

As students master fundamental math concepts and skills, they must also develop appropriate problem-solving skills and strategies. In *Vmath* students: 1) use a problem-solving plan, 2) develop problem-solving strategies, 3) participate in problem-solving strategy practice, and 4) practice of problem-solving applications. Explicit instruction in these problem-solving techniques is consistent with results of research studies examined in a meta-analysis of research on instructional strategies for learning disabled students (Gersten, Chard, Baker and Lee, in review).

1) **Using a Problem-Solving Plan:** The problem-solving plan in *Vmath* incorporates the four phases of the problem-solving plan devised by George Polya in his book How to Solve It (Polya, 1957). In the Get Started section of the lesson the teacher guides the students through the process of understanding the problem, making a plan, carrying out the plan, and looking back to review. In the Try it Together section, the student is directed to ask questions such as "What am I trying to find?", "How can I find the answer?", "What operation should I use?", and "Why is my answer correct?" Students independently practice using a problem-solving plan in the Work On Your Own and the Check Up sections of the lesson.
2) **Developing Problem-Solving Strategies:** *Vmath* teaches the basic strategies that struggling math students should be able to apply in the context of problem-solving. The strategies include:

- Using a Problem-Solving Plan
- Using Reasonableness
- Drawing a Picture
- Finding a Pattern
- Using Guess and Check
- Using a Table
- Using a Graph
- Solving a Simpler Problem
- Using Logical Reasoning
- Using a Diagram
- Choosing an Operation
- Working Backward
- Using Estimation
- Writing an Equation
- Making a Model

The curriculum must simultaneously develop conceptual understanding, fluency in computation and operations, and problem solving skills.

Using the direct, explicit, and systematic format of *Vmath*, the teacher models the critical thinking skills and unique procedures in learning and applying each strategy.

3) **Scaffolded Problem-Solving Strategy Practice:** *Vmath* provides scaffolded problem-solving strategy practice in the Work On Your Own section of alternate lessons. In the "Problem-Solving" box students are guided through the use of a specific problem-solving strategy. Later they will apply the strategy independently as they practice on their own.

4) **Practicing Problem-Solving Applications:** The final question in the Check Up section is an open-ended question that provides the student with an opportunity to apply the problem-solving strategies of Estimation and Checking for Reasonable Answers.

   Students continue to use their problem-solving skills as they apply higher-order thinking skills in the Critical and Algebraic Thinking Problems. Students demonstrate their understanding as they apply their reasoning skills and justify their thinking in the Explain It and Write Math Problems.

The use of formative assessment improves students' learning, especially if teachers have additional guidance on using the assessment to design and individualize instruction.

Vmath Assessment

Researchers have found that the "use of formative assessment improves students' learning, especially if teachers have additional guidance on using the assessment to design and individualize instruction." (National Mathematics Advisory Panel Report, 2008) *Vmath* provides a full range of assessments to enhance instructional decisions, including Initial and Final Assessments, Progress Assessments, powered by the Quantile Framework® for Mathematics, module specific Pre- and Post-Tests, Curriculum-Based Measurements, and frequent opportunities for ongoing assessment. Research suggests that the practice of providing feedback on student performance as well as information related to a student's specific strengths and weaknesses is a combination beneficial to student achievement (Gersten, Chard, Baker and Lee, in review).

Initial and Final Assessments

The *Vmath* Initial Assessment is a formative assessment that is given at the beginning of the level and provides teachers with the information they need to determine a student's strengths and weaknesses. Results gathered from this assessment are reported in a format that allows teachers to determine an effective course of intervention for each student.

The *Vmath* Final Assessment is given at the end of the level. By comparing scores from the Initial and Final Assessments, teachers are able to document student growth and mastery over the course of intervention.

Progress Assessments Powered by the Quantile Framework for Mathematics

The Quantile Framework, developed by MetaMetrics, consists of a common supplemental metric—the Quantile—that is employed to scientifically measure students' performance in thinking mathematically and mathematics achievement and locate them in a taxonomy of mathematical skills, concepts, and applications. Both student achievement and mathematical instructional materials can be measured on the Quantile Framework. When the Quantile level of the instructional materials and student achievement match, it is possible to forecast student success on the daily lesson.

The *Vmath* Progress Assessments are based on the Quantile Framework for Mathematics and are used to measure a student's readiness to learn as well as monitor student progress toward grade-level goals. Using the information gathered from these assessments, teachers are able to determine the likelihood of student's understanding and differentiate instruction to increase student success.

Pre-Tests and Post-Tests

The *Vmath* Pre-tests and Post-Tests focus on the instructional content included in each module. Using these assessments teachers are able to monitor growth, assess mastery of the specific content taught over a limited period of time, and determine if reteaching is needed.

- Pre-Tests determine student's prior knowledge of content and help teachers establish instructional priorities over that specific content.
- Post-Tests determine students' degree of mastery of content after instruction, and help teachers determine if skills have been mastered, or if review or reteaching may be needed.

Curriculum-Based Measurement

The scientific research base used for developing the Curriculum-Based Measurements was initiated by a group of educational scientists headed by Dr. Stanley Deno at the University of Minnesota (Deno, 1985). Their research led to a system of measures later called Curriculum-Based Measurement (CBM) that sought to identify reliable and valid ways of assessing students' progress in essential mathematical skills. Research studies have shown that progress monitoring can improve student performance (Baker et al, 2002; Gersten et al, in review). CBM has been shown to discriminate between students who achieve typically and those in compensatory programs (Deno, Fuchs, Marston, and Shinn, 2001).

Research studies have shown that progress monitoring can improve student performance.

Furthermore, Marston, (1989) and Thurber, Shinn and Smolkowski, (2002) have shown that having students write answers to grade-level computational problems for a short interval of time is a reliable and valid outcome measure of general mathematics computation for typically achieving students through grade 6. Fuchs et al, (1994) found that providing information and instructional suggestions to teachers about student performance positively affected student performance. *Vmath* provides both types of information to teachers.

Vmath Benchmark and Progress Monitoring Computational Fluency Assessments

The *Vmath* Benchmark and Progress Monitoring Assessments are Curriculum Based Measures that focus primarily on fluency in operations and processes. These assessments include three benchmark assessments, ten progress monitoring assessments, and a data management system.

For each *Vmath* level, an assessment prototype is constructed in three steps. First, the computational problem types are identified by examining the computational objectives for that level. Second, these problem types are weighted rationally to reflect their importance to success in mathematics computation. Third, the weighted problem types are randomized to equalize the order of the problem types on each *Vmath* Benchmark and Progress Monitoring Computational Fluency Assessment. Once the prototype is complete, individual items are generated for all assessments.

The result is a mapping of equivalent problems in the same position for each of the thirteen assessments. For example, if the first problem in the first row is adding two three-digit numbers containing two decimal places with an answer containing 4 digits, the first problem in each of the thirteen assessments will fulfill this same requirement. Thus, all tests are equivalent forms allowing for measurement of growth over time. These forms are brief screening measures that focus on critical math skills strongly predictive of future mathematical growth and success. CBM results can be used to indicate how well a student is progressing throughout the entire grade level, and over time the results become good predictors of grade level success. One specific finding by Gersten, Chard, Baker, and Lee (in review) was that providing teachers with specific information on student performance enhanced achievement. The *Vmath* assessments are designed to provide this type of feedback to teachers.

Ongoing Assessment

In Vmath, teachers observe student performance and monitor student responses using detailed teaching dialog, appropriate questioning techniques, and specific correction procedures. In the Get Started section the teacher monitors understanding of pre-skills and new skills. During the Try it Together section the teacher gauges student readiness to move to independent practice and uses correction procedures as needed. Ongoing assessment continues in the Work On Your Own section as the teacher checks students' daily work and uses correction procedures as needed.

Error Analysis

A critical element of explicit instruction for students with mathematical difficulties or the lowest third of the class, is immediate, extensive feedback. (National Mathematics Advisory Panel Final Report, 2008) The Check Up section of each lesson provides the teacher with a daily opportunity to analyze the errors made by each student, determine the exact cause of the error, and apply appropriate correction procedures to remediate and eliminate the error.

A critical element of explicit instruction for students with mathematical difficulties is immediate, extensive feedback

Online Data Management System

Research shows strong support for the use of progress monitoring to improve student performance. (Baker et al, 2002, Gerstein et al, under review) *Vmath* includes an online data management system called VPORT® to help teachers monitor student progress throughout the level. Through the use of VPORT, teachers are able to gather, manage, and report real-time data based on student results on the *Vmath* assessments. This data provides the information teachers need to make effective instructional decisions to improve student success. Utilizing the data management system, teachers can: 1) identify student needs, 2) evaluate student learning, 3) monitor student progress, and 4) differentiate instruction.

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